
GyroSplat: A Linear Representation of Fusion Plasma Turbulence with Direct and Differentiable Physics

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Abstract

Gyrokinetic simulations model the plasma turbulence governing heat transport in magnetically confined fusion devices. These 5D simulations are computationally expensive and produce massive data volumes, with a single snapshot requiring tens of gigabytes of storage. This makes directly storing intermediate timesteps impractical and motivates the use of (lossy) compression. Recent neural compression approaches achieve favorable rate–distortion trade-offs by augmenting conventional L_2 reconstruction objectives with physics-informed losses that preserve derived quantities such as the electrostatic potential and heat flux. However, neural network based representations can be difficult to optimize and refine when additional constraints are imposed. We introduce GyroSplat, a compression method for gyrokinetic fields inspired by Gaussian splatting. GyroSplat represents each plasma field as a weighted sum of independent Gaussian–Gabor atoms, yielding a representation that is linear in its coefficients. This linearity enables a staged optimization strategy in which atoms are first fitted to minimize reconstruction error, then refined against the potential, and subsequently new atoms are added to correct the transport. Moreover, relevant derived quantities admit closed-form expressions in terms of the atom coefficients, enabling efficient refinement via a Gauss–Newton procedure. Experiments on gyrokinetic simulation data show that GyroSplat achieves improved trade-offs at matched storage over established compression baselines on derived physical quantities while remaining competitive on field reconstruction, and improving over state-of-the-art physics-informed compressors. These results demonstrate Gaussian splatting as an effective and flexible approach to high-fidelity compression of scientific simulation data.

1 Introduction

Gyrokinetic simulations are widely used to study turbulent transport in magnetically confined fusion plasmas, which remains a central challenge for commercial fusion energy. These 5D simulations are computationally expensive and produce massive data volumes, making it impractical to retain intermediate states for subsequent use. Lossy compression provides the means for reducing storage requirements while preserving scientifically relevant downstream quantities [4, 12].

Adding physical constraints, however, can result in instabilities during optimization [21, 37]. In neural representations, physical constraints are coupled through shared parameters and they enter as a loss term on the network rather than an algebraic property of the stored code, making targeted refinement difficult [24]. This motivates expressive compression representations that are straightforward to optimize as additional physical constraints are introduced.

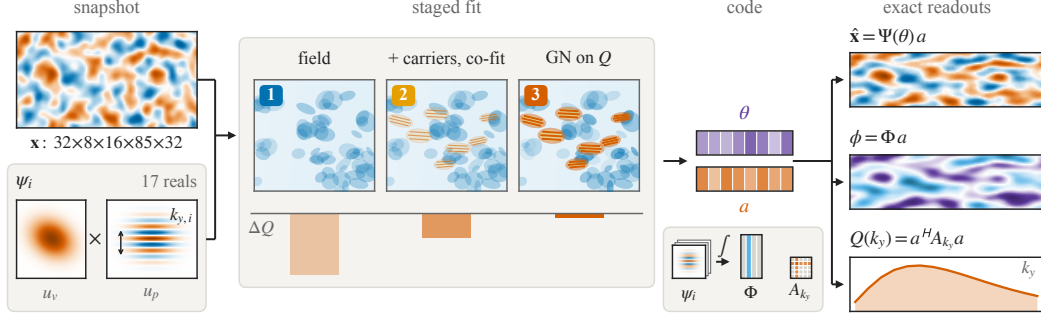


Figure 1: The GyroSplat pipeline. One snapshot is fitted by N Gaussian–Gabor atoms (bottom left: 17 reals each, a velocity envelope, a spatial envelope and a carrier at $k_{y,i}$; §2). The fit is staged: the field first, then carrier atoms with the masked co-fit to correct physicality, then a Gauss–Newton polish on $Q(k_y)$ itself (ΔQ is schematic and the fields are illustrative). The stored code is the geometry θ and the amplitudes a . Each atom is rendered once through the integral chain (bottom right) and the stacked responses form Φ and A_{k_y} . Unlike the full field, which is reconstructed from the atoms, the potential and flux are then read off directly as the closed-form identities $\phi = \Phi a$ and $Q(k_y) = a^H A_{k_y} a$ (§2, App. B).

We introduce *GyroSplat*, a compression method for gyrokinetic fields inspired by Gaussian splatting. GyroSplat represents a snapshot as a weighted sum of independent Gaussian–Gabor atoms [39], yielding a flexible, spatially localized representation that is linear in its coefficients. This structure enables staged optimization. First atoms are fitted for field reconstruction, then refined against the potential, and finally additional atoms can be introduced to refine for physics. Because each atom is independent and has local support, the 5D field, electrostatic potential, and flux terms can be assigned to disjoint groups of atoms, so that refining one observable does not trade against the others. Moreover, important gyrokinetic observables admit direct, closed-form expressions in this representation, without rendering the field, enabling efficient physics-aware refinement with a Gauss–Newton procedure.

Experiments on gyrokinetic simulation data show that GyroSplat achieves improved trade-offs at matched storage over established baselines for both field reconstruction and derived physical quantities. In particular, refining the representation for physical fidelity substantially improves quantities such as heat flux while maintaining state-of-the-art reconstruction quality.

Our contributions are:

- We introduce *GyroSplat*, a flexible Gaussian–Gabor representation for gyrokinetic fields with a staged optimization strategy. Its linear structure and closed-form physical observables enable efficient physics-aware refinement using a Gauss–Newton procedure.
- We demonstrate that GyroSplat improves compression–fidelity trade-offs over established baselines for both field reconstruction and derived physical quantities.

2 Gaussian–Gabor atoms and their direct readouts

A snapshot stores the complex distribution function $\mathbf{x} \in \mathbb{C}^{v_{\parallel} \times \mu \times s \times x \times y}$ on a five-dimensional grid, with axes ordered as the velocity along the field line $v_{\parallel}$ (32 points), the magnetic moment μ (8), the position along the field line s (16), the radial coordinate x (85), and the toroidal coordinate y .

A Gaussian splat is a smooth contribution concentrated around a chosen location. Following the 3D Gaussians used by Kerbl et al. [18] defined the unscaled splat’s envelope g as

$$g(p; m, L) = \exp\left[-\frac{1}{2}\|L^{-1}(p - m)\|_2^2\right], \quad (1)$$

where p is a position, m is the center, and $LL^{\top}$ is the covariance controlling width and orientation. Each splat i has a complex amplitude a_i which controls the magnitude and phase of its contribution to the reconstructed field. The final reconstruction is a weighted sum over all splats. Note that in contrast to scene rendering, there is no opacity value.

The gyrokinetic distribution depends on position $u_p = (s, x, y)$ and velocity $u_v = (v_{\parallel}, \mu)$. Here, s follows the magnetic field line, x crosses the nested flux surfaces, y is the binormal coordinate, and $v_{\parallel}$ and μ denote the parallel velocity and magnetic moment. We factorize the two coordinate groups into $u = (u_v, u_p)$ and form a 5D envelope as

$$g_i(u) = \underbrace{g(u_v; m_i^v, L_i^v)}_{\text{velocity envelope}} \underbrace{g(u_p; m_i^p, L_i^p)}_{\text{spatial envelope}}. \quad (2)$$

The factors L_i^v, L_i^p are lower triangular with positive diagonals, ensuring valid covariance matrices. Each splat is separable between velocity and position. Their sum can describe correlations because splats at different positions can have different velocity centers and widths, giving a velocity distribution that changes with spatial position.

To represent oscillations at frequencies which are relevant to heat transport, we multiply the Gaussian envelope by a plane wave, following Gabor representations [11, 39]. The resulting Gaussian–Gabor splat and the decoded distribution are

$$\psi_i(u) = g_i(u) e^{ik_i^T (u_p - m_i^p)}, \quad \hat{f}(u) = \sum_{i=1}^N a_i \psi_i(u). \quad (3)$$

Here N is the number of splats, $a_i \in \mathbb{C}$ is a complex amplitude, and k_i is the spatial frequency of the oscillatory factor. We call the Gaussian with $k_i = 0$ a *base splat* and the modulated Gaussian with $k_i \neq 0$ a *Gabor splat*. The real part of a Gabor splat alternates between positive and negative values within its envelope. Modulation shifts the spectrum toward the chosen frequency, allowing one localized splat to describe several oscillations. We modulate along the binormal direction, $k_i = (0, 0, k_{y,i})$, because it determines large parts of the turbulent transport.

An independently stored splat has five center coordinates, nine Cholesky entries, one modulation frequency and two amplitude values, giving 17 real parameters. We deploy the Gabor splats as groups, where the 14 envelope parameters are shared and stored once per group, while their individual amplitudes and frequencies remain separate. This merging of Gabor splats into groups enhances the storage capabilities by more than three times, compared to base splats. A Gabor group occupies the same location, but with the learnable amplitudes can adjust their influence on the reconstruction. We also store four normalization values per snapshot.

Readouts. A readout is the map from the stored code to a derived quantity. Splitting the code into a geometry part $\theta = \{(m_i, L_i, k_{y,i})\}$ and an amplitude vector $a \in \mathbb{C}^N$ makes the decode linear once the geometry is frozen, $\hat{\mathbf{x}} = \Psi(\theta)a$ (App. Fig. 1). Both observables are simple functions of the underlying field: quasineutrality maps the field linearly to the electrostatic potential [3], and the heat flux correlates the field with the drift generated by that potential, making it quadratic. A linear decode therefore inherits both structures. The potential becomes $\phi = \Phi a$, where $\Phi \in \mathbb{C}^{43,520 \times N}$ has one row per point of the spatial grid (s, x, y) and column i holds the potential field of atom i at unit amplitude. The flux becomes $Q(k_y) = a^H A_{k_y} a$, with one Hermitian $N \times N$ matrix per flux mode whose entry (i, j) is the flux carried by the atom pair (i, j) . Both operators remain constant at fixed geometry (for a step-by-step derivation see App. B).

The operators are assembled numerically once per frozen geometry: each atom is rendered individually (a full 5D tensor at unit amplitude) and passed through the linear render-and-solve chain a single time, and Φ and A are assembled from these per-atom responses (App. A). After this assembly, evaluating the flux requires only a quadratic form and its gradient a single matrix–vector product (App. A). The same operators can be assembled for any representation whose decode is linear in its coefficients, and §3 uses this to fit a Fourier and a Haar representation with the same objective (Table 1).

Fitting. Encoding a snapshot is an optimization problem, which we solve in stages. First base atoms are initialized with $k_{y,i} = 0$, which are fitted to minimize reconstruction error. In this step the geometry is optimized by gradient descent on a field loss, while the amplitudes are solved directly at every step (via variable projection [14]). Afterwards, the potential term is applied on the base atoms (similar to the 2nd-stage neural-network optimization in PINC [12]). Next, new atoms are added to correct the transport. These added carrier atoms ($k_{y,i} > 0$) give the flux spectrum explicit support and act as the selection mask for the flux-aware stages. The carriers must be deployed

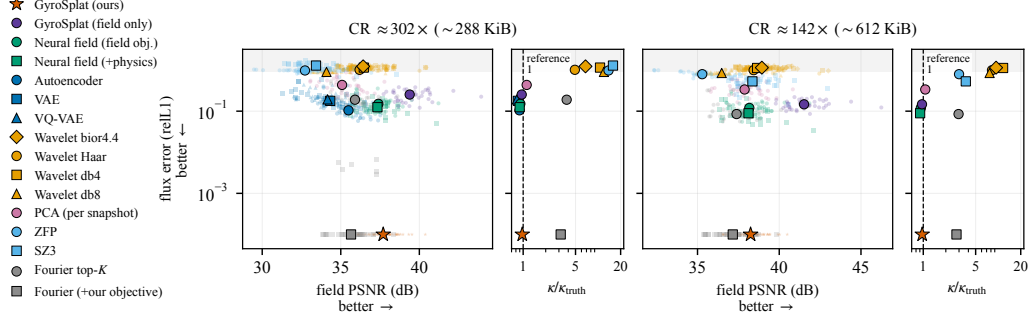


Figure 2: **Direct readouts in the fitting loss hold the physics that field-fitted methods lose.** Wide panels: field PSNR against flux error at the two compression ratios (CR); light points are the 39 snapshots, solid markers method medians (legend left). Methods fitted to the field alone spread across the upper flux band regardless of field score. GyroSplat, and the Fourier code fitted through the same readouts, hold the flux error below 10^{-4} . Slim panels: the same medians against their cancellation ratio κ , the gross transport per unit of net (truth 1.89; defined in §3); GyroSplat sits at the truth’s 1.89 while the Fourier control carries 5.75 and 4.96.

deliberately, as backpropagation does not give a strong enough signal to change the base atoms carrier class $k_{y,i} = 0$ (App. C). A masked co-fit then brings the flux supervision in, directing the field and potential gradients to the base atoms and the flux gradient to the carriers around a ϕ -aware direct amplitude solve. Finally, a Gauss–Newton polish [29] refines the amplitudes against $Q(k_y)$ itself (App. C). The complete fit takes 271 s at $N=9216$ on a single GH200 (App. A).

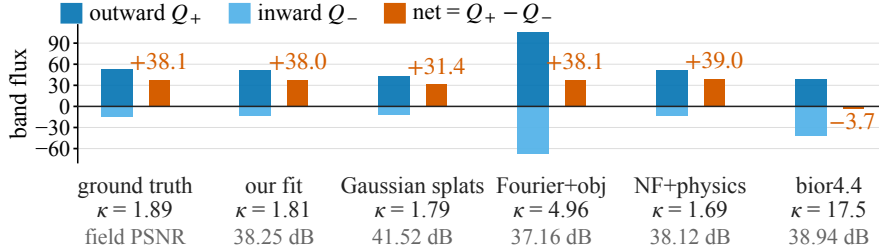


Figure 3: Signed band transport of one snapshot (CR of 142×), net in orange, with each group’s median κ and field PSNR below (Tables 2 and 1). Ours matches the truth’s distribution function, the field-only fit undershoots the net, the objective-fitted Fourier fits the net while keeping too much gross transport, and bior4.4 flips it (all methods: Fig. 4).

3 Results

We evaluate GyroSplat against twelve baselines on 39 evaluation snapshots at both compression ratios. The baselines cover classical codecs (the wavelets, PCA, ZFP, SZ3, and Fourier top- K), neural-network based compressors (autoencoder (AE), variational autoencoder (VAE) and vector-quantized autoencoder (VQ-VAE); setup from PINC [12] but trained only on reconstruction), per-snapshot neural fields (NF) with and without the physics stage, and GyroSplat fitted to the field alone. All methods are evaluated on four quantities. PSNR (peak signal-to-noise ratio) for the field reconstruction, potential PSNR, the relative error of the flux spectrum, and the cancellation ratio $\kappa = (Q_+ + Q_-)/|Q|$, with $Q_{\pm}$ the outward and inward parts of the transport. The two nearly cancel, so κ measures how much gross transport a reconstruction carries per unit of net. The ground truth’s κ is 1.89, and values far above it mean a reconstruction reaches its net by carrying transport the data do not have (protocol details, budgets, and every baseline’s recipe App. A).

We report results in Table 2 (App. A). GyroSplat holds the flux error below 10^{-4} , achieves the best potential PSNR in the comparison, and remains competitive on field reconstruction. Compared with the physics-informed neural field, the state of the art for this data, it is ahead on every column at

Table 1: **The fitting objective transfers to other linear representations, and GyroSplat uses it best.** Each row fits a different basis with the same staged objective at the same CR (App. A). All three pin the flux below 10^{-4} (bold ties); the support decides everything else. **Bold** marks the best column entry per budget; κ closest to the truth’s 1.89 is bold.

representation	PSNR _f ↑	PSNR _ϕ ↑	flux relL1 ↓	κ (truth 1.89)
<i>CR ≈ 302×</i>				
Haar (+ our objective)	16.60	−8.50	< 10^{-4}	14.5
Fourier (+ our objective)	35.65	23.59	< 10^{-4}	5.75
GyroSplat (ours)	37.70	27.30	< 10^{-4}	1.82
<i>CR ≈ 142×</i>				
Haar (+ our objective)	26.40	−6.94	< 10^{-4}	5.82
Fourier (+ our objective)	37.16	27.11	< 10^{-4}	4.96
GyroSplat (ours)	38.25	28.71	< 10^{-4}	1.81

both budgets, with the largest gap on the flux, where the neural field reads 0.1249 and 0.0890 while GyroSplat stays below 10^{-4} . Methods without a physics term do not control the flux, as we show their errors span 0.085 to 1.29, and the two best-field wavelet bases reverse the sign of the net flux on 38–39 of the 39 snapshots while scoring among the best field PSNRs (per-basis counts in App. A; Fig. 3 shows one such snapshot, where the Wavelet bior4.4 turns outward transport inward). This is possible because the net flux is a small difference of large outward and inward contributions that nearly cancel, so a reconstruction can be close to the field everywhere and still lose the balance with almost no change in its field score.

We fit a Fourier and a Haar representation with the same staged objective at the same CR, and both reach the flux floor. However, the Fourier basis reaches it at $2.6\times$ the truth’s gross transport (κ 5.0 vs 1.89) and the Haar basis at a 17 dB loss of field quality, whereas GyroSplat reaches it without either cost. The staged refinement can be applied to any representation that decodes linearly, but its cost depends on the representation, and GyroSplat performs best. The staging is itself measured in the ablation table of App. C. Turning the flux term on already at the potential stage, instead of after the carriers are added, is worse on every column including the flux (0.42 against 0.28), and the Gauss–Newton step takes the flux median from 10^{-4} after the co-fit to about 2×10^{-7} (App. A).

4 Takeaways

Physics survives compression when it is a property of the code. Because the representation is linear in its coefficients, the potential and the heat flux remain direct algebraic readouts of the stored atoms, and the staged optimization can refine them directly, without trading one observable against another. Refining for physical fidelity substantially improves the derived quantities at matched storage, while field reconstruction remains competitive and a field score alone predicts neither derived quantity. Once the direct readout is the target, a low flux error says only that the supervision worked. The cancellation ratio κ is what still separates the representations (§3).

Limitations. Our potential and flux values are fitted through the direct readouts, while every baseline except the neural field’s physics stage optimizes the field alone. Before its flux stage, our own fit stands at flux 0.25/0.18, worse than the unsupervised Fourier baseline’s 0.085, and the potential margin over the neural field is a median claim, as the quartiles overlap (Table 6). Two controls bound the mechanism. A field-optimal amplitude solve collapses the potential by 18.6 dB (App. C), and refining against wrong-snapshot targets leaves the potential 14.5 dB below the no-refinement floor (App. A). Direct readouts also do not mean exact reconstructions. The readout computes the potential and the flux of the stored atoms up to the numerical agreement of the assembled operators (10^{-6} – 10^{-5} , App. A), and how close those are to the truth still depends on how well GyroSplat fits the snapshot. Finally, GyroSplat has no encoder network, so every snapshot requires its own optimization (§2).

Practices. Two practical lessons follow. Compressed simulation data should be evaluated on the derived quantities next to the field metric, because the net flux is a small residual of two large opposing contributions and a good field score does not show whether that residual is preserved. And when the representation has support on the relevant modes, adding the direct readouts to the fitting loss costs no additional storage and reduces the flux error by orders of magnitude (§3).

References

- [1] Mark Ainsworth, Ozan Tugluk, Ben Whitney, and Scott Klasky. Multilevel techniques for compression and reduction of scientific data-quantitative control of accuracy in derived quantities. *SIAM Journal on Scientific Computing*, 41(4):A2146–A2171, 2019. doi: 10.1137/18M1208885.
- [2] G Berkooz, P Holmes, and J L Lumley. The proper orthogonal decomposition in the analysis of turbulent flows. *Annual Review of Fluid Mechanics*, 25:539–575, 1993. doi: 10.1146/annurev.fl.25.010193.002543.
- [3] A. J. Brizard and T. S. Hahm. Foundations of nonlinear gyrokinetic theory. *Rev. Mod. Phys.*, 79:421–468, Apr 2007. doi: 10.1103/RevModPhys.79.421.
- [4] Jong Choi, Michael Churchill, Qian Gong, Seung-Hoe Ku, Jaemoon Lee, Anand Rangarajan, Sanjay Ranka, Dave Pugmire, CS Chang, and Scott Klasky. Neural data compression for physics plasma simulation. In *Neural Compression Workshop @ ICLR*, 2021.
- [5] Albert Cohen, Ingrid Daubechies, and Jean-Christophe Feauveau. Biorthogonal bases of compactly supported wavelets. *Communications on Pure and Applied Mathematics*, 45(5):485–560, 1992. doi: 10.1002/cpa.3160450502.
- [6] Ingrid Daubechies. *Ten Lectures on Wavelets*, volume 61 of *CBMS-NSF Regional Conference Series in Applied Mathematics*. SIAM, 1992.
- [7] Ronald A. DeVore. Nonlinear approximation. *Acta Numerica*, 7:51–150, 1998. doi: 10.1017/S0962492900002816.
- [8] Landon Dyken, Sharmistha Chakrabarti, Nathan Debardeleben, Steve Petruzza, Qi Wu, Will Usher, and Sidharth Kumar. Efficient compression of structured and unstructured volumes via learned 3d gaussian representation, 2026.
- [9] Carl Eckart and Gale Young. The approximation of one matrix by another of lower rank. *Psychometrika*, 1(3):211–218, 1936. doi: 10.1007/BF02288367.
- [10] Lukas Einkemmer, Katharina Kormann, Jonas Kusch, Ryan G. McClarren, and Jing-Mei Qiu. A review of low-rank methods for time-dependent kinetic simulations. *Journal of Computational Physics*, 538:114191, 2025. ISSN 0021-9991. doi: 10.1016/j.jcp.2025.114191.
- [11] Dennis Gabor. Theory of communication. part 1: The analysis of information. *Journal of the Institution of Electrical Engineers - Part III: Radio and Communication Engineering*, 93: 429–441, 1946. doi: 10.1049/ji-3-2.1946.0074.
- [12] Gianluca Galletti, Gerald Gutenbrunner, Sandeep S. Cranganore, William Hornsby, Lorenzo Zanisi, Naomi Carey, Stanislas Pamela, Johannes Brandstetter, and Fabian Paischer. Physics-informed neural compression of high-dimensional plasma data, 2026. arXiv:2602.04758.
- [13] Gianluca Galletti, Gerald Gutenbrunner, William Hornsby, Lorenzo Zanisi, Naomi Carey, Stanislas Pamela, Johannes Brandstetter, and Fabian Paischer. A shortcut to statistically steady-state turbulence with flow matching, 2026. arXiv:2607.13022.
- [14] G. H. Golub and V. Pereyra. The differentiation of pseudo-inverses and nonlinear least squares problems whose variables separate. *SIAM Journal on Numerical Analysis*, 10(2):413–432, 1973. doi: 10.1137/0710036.
- [15] Bo Jiang, Youyuan Liu, Taolue Yang, Sheng Di, and Sian Jin. 3d gaussian splatting for scientific particle data compression and rendering, 2026.
- [16] Pu Jiao, Sheng Di, Hanqi Guo, Kai Zhao, Jiannan Tian, Dingwen Tao, Xin Liang, and Franck Cappello. Toward quantity-of-interest preserving lossy compression for scientific data. *Proc. VLDB Endow.*, 16(4):697–710, December 2022. doi: 10.14778/3574245.3574255.

- [17] Namgyu Kang, Jaemin Oh, Youngjoon Hong, and Eunbyung Park. Pig: Physics-informed gaussians as adaptive parametric mesh representations. In Y. Yue, A. Garg, N. Peng, F. Sha, and R. Yu, editors, *International Conference on Learning Representations*, volume 2025, pages 73022–73046, 2025.
- [18] Bernhard Kerbl, Georgios Kopanas, Thomas Leimkuehler, and George Drettakis. 3d gaussian splatting for real-time radiance field rendering. *ACM Transactions on Graphics*, 42(4), July 2023. doi: 10.1145/3592433.
- [19] Diederik P. Kingma and Jimmy Ba. Adam: A Method for Stochastic Optimization. In *International Conference on Learning Representations*, 2015.
- [20] Robert H. Kraichnan. Inertial ranges in two-dimensional turbulence. *The Physics of Fluids*, 10(7):1417–1423, 07 1967. doi: 10.1063/1.1762301.
- [21] Aditi Krishnapriyan, Amir Gholami, Shandian Zhe, Robert Kirby, and Michael Mahoney. Characterizing possible failure modes in physics-informed neural networks. In M. Ranzato, A. Beygelzimer, Y. Dauphin, P.S. Liang, and J. Wortman Vaughan, editors, *Advances in Neural Information Processing Systems*, volume 34, pages 26548–26560. Curran Associates, Inc., 2021.
- [22] Xin Liang, Kai Zhao, Sheng Di, Sihuan Li, Robert Underwood, Ali M. Gok, Jiannan Tian, Junjing Deng, Jon C. Calhoun, Dingwen Tao, Zizhong Chen, and Franck Cappello. Sz3: A modular framework for composing prediction-based error-bounded lossy compressors. *IEEE Transactions on Big Data*, 9(2):485–498, 2023. doi: 10.1109/TBDATA.2022.3201176.
- [23] Peter Lindstrom. Fixed-rate compressed floating-point arrays. *IEEE Transactions on Visualization and Computer Graphics*, 20(12):2674–2683, 2014. doi: 10.1109/TVCG.2014.2346458.
- [24] Qiang Liu, Mengyu Chu, and Nils Thuerey. Config: Towards conflict-free training of physics informed neural networks. In *International Conference on Learning Representations*, volume 2025, pages 59531–59566, 2025.
- [25] Ilya Loshchilov and Frank Hutter. Decoupled Weight Decay Regularization. In *International Conference on Learning Representations*, 2019.
- [26] S.G. Mallat and Zhifeng Zhang. Matching pursuits with time-frequency dictionaries. *IEEE Transactions on Signal Processing*, 41(12):3397–3415, 1993. doi: 10.1109/78.258082.
- [27] Mohammadreza Momenifar, Enmao Diao, Vahid Tarokh, and Andrew D. Bragg. A physics-informed vector quantized autoencoder for data compression of turbulent flow. In *2022 Data Compression Conference (DCC)*, pages 01–10, 2022. doi: 10.1109/DCC52660.2022.00039.
- [28] Thomas Müller, Alex Evans, Christoph Schied, and Alexander Keller. Instant neural graphics primitives with a multiresolution hash encoding. *ACM Transactions on Graphics*, 41(4), July 2022. doi: 10.1145/3528223.3530127.
- [29] Jorge Nocedal and Stephen J. Wright. *Numerical Optimization*. Springer Series in Operations Research and Financial Engineering. Springer, 2nd edition, 2006.
- [30] Fabian Paischer, Gianluca Galletti, William Hornsby, Paul Setinek, Lorenzo Zanisi, Naomi Carey, Stanislas Pamela, and Johannes Brandstetter. Gyroswin: 5d surrogates for gyrokinetic plasma turbulence simulations. In D. Belgrave, C. Zhang, H. Lin, R. Pascanu, P. Koniusz, M. Ghassemi, and N. Chen, editors, *Advances in Neural Information Processing Systems*, volume 38, Main Conference, pages 49227–49254. Curran Associates, Inc., 2025. doi: 10.52202/085713-1647.
- [31] A. G. Peeters, Y. Camenen, F. J. Casson, W. A. Hornsby, A. P. Snodin, D. Strintzi, and G. Szepesi. The nonlinear gyro-kinetic flux tube code GKW. *Computer Physics Communications*, 180(12):2650–2672, 2009. doi: 10.1016/j.cpc.2009.07.001.
- [32] M. L. Rensen. Physics-informed Gaussian splatting: Solving partial differential equations with Gaussians. Master’s thesis, Delft University of Technology, 2024. MSc thesis, Computer Graphics and Visualisation group; supervisors M. Weinmann, B. Buschmann, E. Eisemann.

- [33] G. Rozza, D. B. Phuong Huynh, and A. T. Patera. Reduced basis approximation and a posteriori error estimation for affinely parametrized elliptic coercive partial differential equations. *Archives of Computational Methods in Engineering*, 15(3):229–275, 2008. doi: 10.1007/s11831-008-9019-9.
- [34] Aleix Seguí and Wesley Armour. A geometric lens on physics-aligned data compression. In *Proceedings of the 43rd International Conference on Machine Learning*, Proceedings of Machine Learning Research. PMLR, 2026.
- [35] Vincent Sitzmann, Julien Martel, Alexander Bergman, David Lindell, and Gordon Wetzstein. Implicit neural representations with periodic activation functions. In H. Larochelle, M. Ranzato, R. Hadsell, M.F. Balcan, and H. Lin, editors, *Advances in Neural Information Processing Systems*, volume 33, pages 7462–7473, 2020.
- [36] Aaron van den Oord, Oriol Vinyals, and Koray Kavukcuoglu. Neural discrete representation learning. In *Proceedings of the 31st International Conference on Neural Information Processing Systems*, page 6309–6318, 2017.
- [37] Sifan Wang, Yujun Teng, and Paris Perdikaris. Understanding and mitigating gradient flow pathologies in physics-informed neural networks. *SIAM Journal on Scientific Computing*, 43(5):A3055–A3081, 2021. doi: 10.1137/20M1318043.
- [38] Haato Watanabe, Kenji Tojo, and Nobuyuki Umetani. 3D Gabor splatting: Reconstruction of high-frequency surface texture using Gabor noise. In *Eurographics 2025 – Short Papers*. The Eurographics Association, 2025. doi: 10.2312/egs.20251040.
- [39] Skylar Wurster, Ran Zhang, and Changxi Zheng. Gabor splatting for high-quality gigapixel image representations. In *ACM SIGGRAPH 2024 Posters*. Association for Computing Machinery, 2024. doi: 10.1145/3641234.3671081. URL <https://doi.org/10.1145/3641234.3671081>.
- [40] Xinjie Zhang, Xingtong Ge, Tongda Xu, Dailan He, Yan Wang, Hongwei Qin, Guo Lu, Jing Geng, and Jun Zhang. GaussianImage: 1000 FPS image representation and compression by 2D Gaussian splatting. In *European Conference on Computer Vision (ECCV)*, volume 15067 of *Lecture Notes in Computer Science*, pages 327–345. Springer Nature Switzerland, 2024. doi: 10.1007/978-3-031-72673-6_18.

Table 2: **Twelve baselines at matched compression.** Median scores over the 39 evaluation snapshots at each compression ratio (CR). Within each CR the rows run from classical codecs to the neural-networks which use physics-informed compression. κ is the gross-to-net cancellation of the flux integrand (truth 1.89; closest is bold). **Bold** elsewhere marks the best column entry; flux entries below 10^{-4} tie. App. A.

method	PSNR _f ↑	PSNR _ϕ ↑	flux relL1 ↓	κ (truth 1.89)
<i>CR ≈ 302 × (73,728 stored reals)</i>				
Wavelet top- <i>K</i> (bior4.4) [5]	36.42	−12.35	1.2317	13.4
Wavelet top- <i>K</i> (Haar) [6]	36.20	−7.95	1.0150	9.22
PCA, per snapshot [9]	35.08	−2.54	0.4348	2.05
ZFP [23]	32.72	−5.61	0.9781	24.0
SZ3 [22]	33.42	−15.34	1.2874	29.3
Fourier top- <i>K</i> [7]	35.90	22.60	0.1899	7.04
Autoencoder [12]	35.49	15.42	0.1048	1.67
VAE [12]	34.33	9.26	0.1783	1.60
VQ-VAE [36, 12]	34.16	12.91	0.1938	1.57
Neural field (field objective) [12]	37.39	14.73	0.1508	1.73
GyroSplat (field objective only)	39.38	8.43	0.2537	1.80
Neural field (+ physics stage) [12]	37.33	26.50	0.1249	1.69
GyroSplat (ours)	37.70	27.30	< 10^{−4}	1.82
<i>CR ≈ 142 × (156,669 stored reals)</i>				
Wavelet top- <i>K</i> (bior4.4)	38.94	−9.86	1.1441	17.5
Wavelet top- <i>K</i> (Haar)	38.44	−7.12	0.9942	15.9
PCA, per snapshot	37.88	1.69	0.3371	1.98
ZFP	35.29	−3.39	0.7941	5.52
SZ3	38.36	−11.93	0.5311	6.81
Fourier top- <i>K</i>	37.39	26.72	0.0850	5.62
Neural field (field objective)	38.17	14.66	0.1199	1.75
GyroSplat (field objective only)	41.52	9.92	0.1469	1.79
Neural field (+ physics stage)	38.12	26.99	0.0890	1.69
GyroSplat (ours)	38.25	28.71	< 10^{−4}	1.81

A Evaluation protocol

Evaluation-set construction. Balanced-39 is a set of 39 held-out *snapshots* (single saturated-phase time slices, not full trajectories) drawn from 27 distinct nonlinear gyrokinetic flux-tube trajectories computed with GKW [31]; each snapshot is the complex distribution function of the kinetic ion species, with the electrons treated adiabatically, stored as interleaved real and imaginary parts in GKW’s normalization over real coordinates (the diagnostics Fourier-transform the periodic binormal axis, whose modes are the k_y of the banded metrics; §2). The classes: 12 streamer (7 trajectories), 12 mixed (11), 15 strong-zonal (11); two trajectories contribute snapshots to two classes (streamer and mixed) at different times, and at most three timesteps/snapshots come from any one trajectory.

Set construction and classification. The set was built by classifying a 309-candidate pool (the dataset’s test split, earlier in/out-of-distribution probes, and 48 further held-out trajectories at several saturated timesteps) with a regime classifier fixed in advance, then balancing per class. The classifier reads each snapshot’s own stored diagnostics. Normalizing the per- k_y power spectrum to unit mass, the zonal fraction is its $k_y=0$ share and the flux peak is the mode of the unit-normalized flux spectrum; a snapshot is strong-zonal at zonal fraction ≥ 0.60 , otherwise streamer if its flux peak sits at mode ≥ 5 , otherwise mixed.

Held-out dataset. None of the snapshots appears in any learned baseline’s training set (AE, VAE, VQ-VAE), and including how we derived the steps for GyroSplat. Every other family fits per snapshot and has no training set. Ramp-phase (pre-saturation) slices were excluded by convention: apparent “streamers” at early times are transients, and saturated streamer states exist only in a minority of the held-out trajectories, which is why the streamer class draws on fewer of them.

Evaluation metrics. Every family in Table 2 is scored on this snapshot set, and the following metric functions evaluate every member:

$$\text{PSNR}_f = 10 \log_{10} \frac{\max_u |\mathbf{x}(u)|^2}{\langle (\hat{\mathbf{x}} - \mathbf{x})^2 \rangle_u}, \quad \text{PSNR}_\phi = 10 \log_{10} \frac{(\max_u \phi(u))^2}{\langle (\hat{\phi} - \phi)^2 \rangle_u},$$

$$\text{relL1}(Q) = \frac{\sum_{k_y \in [1,16]} |\hat{Q}(k_y) - Q(k_y)|}{\sum_{k_y \in [1,16]} |Q(k_y)|}, \quad \kappa = \frac{Q_+ + Q_-}{|Q|},$$

with $\mathbf{x}$ the stored re/im-interleaved reference field, ϕ the reference potential, $(\hat{\cdot})$ the reconstruction’s readouts, and $\langle \cdot \rangle_u$ the mean over the full grid, and $Q_\pm$ the gross outward and inward parts of the banded transport integrand, $Q = Q_+ - Q_-$ (their construction: the flux-integrand paragraph below). The band carries a median 97% of the reference’s total $|Q|$ mass (93.6–98.5% across the evaluation set; the $k_y=0$ zonal mode contributes none). Dispersion and per-class medians, recomputed from the raw per-snapshot scores (all printed medians reproduce the table): Tables 6–7.

Budgets. CR is the stored-reals ratio against the raw snapshot’s 22,282,240 reals; the block budgets are 73,728 stored reals at $302\times$ and 156,669 at $142\times$. The analytic rows print the written file’s stored count, within four reals of the nominal $17N$. Wavelet and PCA methods record a compression ratio rather than a real count, reached per snapshot. The per-row ratios sit within $301.5\text{--}304.8\times$ and $142.0\text{--}144.9\times$ of the nominal budgets. The convention counts *reals*, not bytes, and the wavelet top- K is per-snapshot adaptive, so its CR is an evaluation-set median rather than a fixed budget. Every family’s printed row stores fp32 reals. Half-precision storage was measured for all of them: recasting the stored values to fp16 after fitting halves the bytes and moves the medians by at most 0.1 dB and 0.008 flux, except the potential column of ours ($-0.15\text{--}0.43$ dB, with flux 0.0018/0.0030), of the wavelets (-0.3 to -1.5 dB), and of PCA and the VAE (-0.6 and -0.4 dB).

Storage accounting for top- K codes. The top- K rows (the wavelets and the Fourier baseline) count coefficient *values* only, while decoding also needs the per-snapshot support map (entropy floor 87 KB at $302\times$, 165 KB at $142\times$; Fourier 82 KB), which puts the wavelets’ true fp32 storage at $232\times/112\times$ where the label reads $302\times/142\times$ ($376\times/185\times$ with fp16 values).

Byte-matched wavelets. Re-run byte-matched, with kept values *plus* support-map floor inside our rows’ total bytes (the Fourier pair’s accounting convention; byte-matched targets $397.4\text{--}398.2\times/181.9\text{--}182.1\times$ per basis by value count; all four bases, 0 gate failures), the wavelets lose 0.6–0.9 dB of field: bior4.4 falls to 35.75 at the $302\times$ budget and to 38.07 at $142\times$, *below* our fit’s 38.25, with the net sign wrong on 39/39 snapshots at both budgets, and the other three bases behave alike (per-basis rows in the artifact). Every printed wavelet claim survives this byte-matched accounting; the field advantages shrink or invert. VQ-VAE’s integer codes are precision-free; our rows are self-contained (atom positions inside the $17N$ budget).

Learned-latent budget. The learned latents (autoencoder, VAE, VQ-VAE) exist only at the 73,728-real budget (the PINC recipe’s latent shape defines it), so the $142\times$ block carries no learned-latent row and nothing is imported across tiers. Additional autoencoder ablations were not run, because of the limits of our computational budget.

Baselines. Learned rows (autoencoder, VAE, VQ-VAE [36]) trained versions, based on the recipe of PINC [12], its plain reconstruction-only autoencoder, VAE, and VQ-VAE configurations without the physics terms, we decided to not trained on the physics terms, based on our computational budget; neural fields are per-snapshot MLP fits at 73,799 and 156,902 reals (within $+0.15\%$ of block budget) under a density objective and with an added physics stage, in the coordinate-network line [35, 28]; wavelet rows are per-snapshot top- K [7] in four bases (Haar; db4 and db8, the compactly supported orthonormal family [6]; bior4.4, the CDF 9/7 biorthogonal pair [5]); Principal component analysis (PCA) is per snapshot. The ZFP rows [23] run a prior protocol of the error-bounded class (compressors that guarantee a per-value error tolerance): one 4D layout per channel pair, with the absolute tolerance searched per snapshot to the target CRs, the stream self-contained bytes. SZ3 [22], the other error-bounded row, is measured the same way; its $142\times$ cell is the comparison’s sharpest split between field and physics: field above our fit (38.36 vs 38.25) with the potential

at -11.9 dB and the net-flux sign kept on all 39 snapshots, against 33/39 wrong at $302\times$. Analytic, wavelet, PCA and ZFP rows are deterministic; learned and NF rows are single checkpoints or single fits, no seed averaging.

Related work. GyroSplat originates from Gabor analysis [11, 39, 38], graphics splatting [18, 40] and matching pursuit [26], while quadratic forms on coefficients are reduced-order machinery [2, 33]. GyroSplat combines the two lines, with observables that are directly accessible in closed-form and therefore cheap to supervise on. The physics-informed line makes them a loss term [27, 4], not a code property. That field error alone can misrank codes, and is itself documented [1, 16, 34]. Physics-informed compression itself is an active line [4, 12], and related neural surrogates exist for the same data [30, 13]. The construction itself requires only that the observable be a linear or quadratic functional of a linearly decoded field, which holds for quantities such as energy or enstrophy in other simulation data [20].

Adjacent lines. Dynamical low-rank reductions for kinetic data [10] remain outside the comparison. Gaussian primitives also already appear as PDE bases [17, 32] and as scientific-data compressors, for scalar-field fidelity [8] and for rendered-image quality [15]; neither line derives direct linear or quadratic observable readouts from the code.

Fitting-seed dispersion. The analytic rows are fit seed 0, fixed in advance. Refitting all 39 snapshots at seven further seeds at the deployed stage (546 fits) puts the per-seed medians at field [37.67, 37.77] and potential [27.09, 27.33] dB at $302\times$, field [38.17, 38.26] and potential [28.62, 28.81] dB at $142\times$, with the flux below 10^{-4} on every seed at both tiers. The printed rows lie within every band. The potential margin over the physics-informed neural field (0.80 dB at $302\times$) sits outside our 0.24 dB seed band and the neural field’s own three-seed band of 0.23 dB (seed 0 plus two explicit-seed refits); the reservation §3 carries is snapshot-level dispersion: the quartile overlap of Table 6.

The flux integrand, $Q_{\pm}$, and κ . The reference diagnostics return the flux *integrand* as a signed field over the full phase-space coordinates $(v_{\parallel}, \mu, s, k_x, k_y)$: at each point, the local energy-weighted fluctuation times the radial $E \times B$ drift: positive where heat is locally carried outward, negative where inward. Summing the signed field gives the deployed net flux (checked to equal the stored value per snapshot); Q_+ is the sum of the positive part, Q_- the magnitude of the negative part, so $Q = Q_+ - Q_-$ and $\kappa = (Q_+ + Q_-)/|Q|$ directly. This *integrand-level* cancellation is invisible to the spectrum: within each k_y bin the two signs have already cancelled, which is why the ground truth’s mode-level ratio $\sum_{k_y} |Q(k_y)| / |\sum_{k_y} Q(k_y)|$ is ≈ 1.0 while its integrand-level κ is 1.89. A field metric sees still less of it.

Cancellation, measured for every family. Per-snapshot gross-to-net κ of the evaluation set’s ground truth (canonical data root, deduplicated to the 39 snapshots): median 1.89, IQR 1.77–1.98. Table 2’s κ column applies the identical banded reduction to every scored reconstruction, each family decoded by its own reference scorer; the two integral chains involved were checked to agree on the ground truth per snapshot, identical on every recorded digit (medians of 39). Every learned code sits *below* the truth (autoencoder 1.67, VAE 1.60, VQ-VAE 1.57, neural fields 1.69–1.75, less gross signed weight per unit of net flux than the data carry), PCA just above (2.05/1.98), the field-objective ablation essentially at the truth’s ratio (1.80/1.79), the Fourier code an order above (5.6–7.0 unsupervised, 5.0–5.8 under our objective), and the top- K and error-bounded codes sit an order of magnitude above it (wavelets 9–23, Table 3; ZFP 24.0/5.5; SZ3 29.3/6.8 with a per-snapshot maximum of 1484). κ is not monotone in the wrong-sign counts (db8 reaches $\kappa=23$ yet flips only 3/39) because κ measures the excess gross weight, while a flip additionally requires the residual net to cross zero. Fig. 4 draws the full anatomy (gross outward, gross inward, net) for the drawn families at one snapshot.

The VAE draw band. The VAE decoder samples its latent at decode time (no deterministic mode in its forward), so every scored VAE row is one decode draw; across five seeded draws the κ and field medians move only 1.59–1.60 and 0.03 dB, while the potential median spans 8.6–11.7 dB and the flux 0.17–0.20; the printed row sits inside both bands.

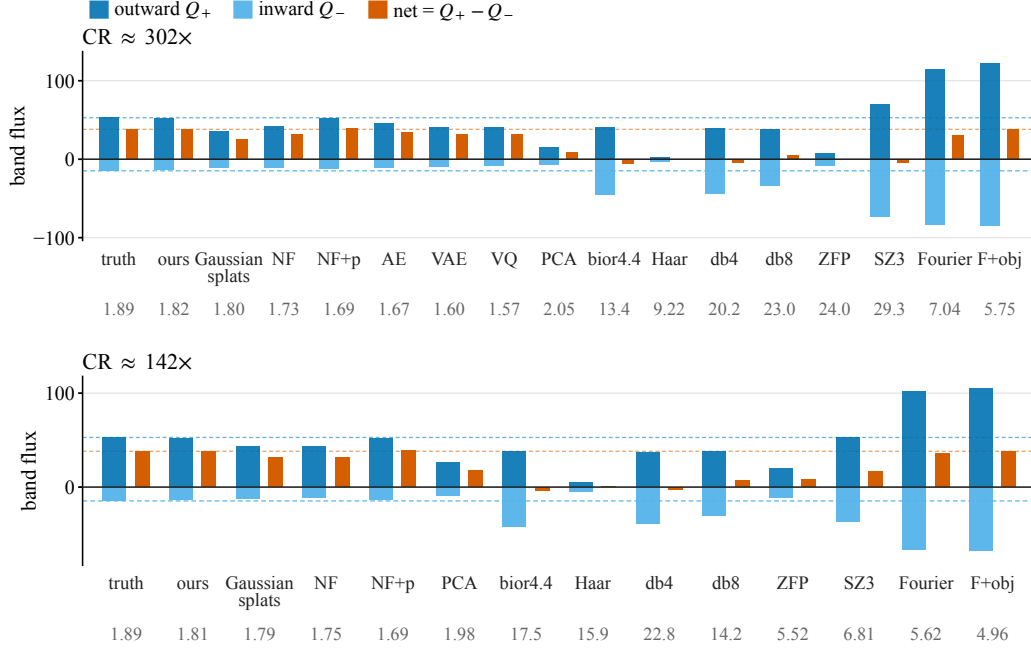


Figure 4: The flux-integrand anatomy of Fig. 3 for the baseline families and the supervised Fourier (the objective-fitted Haar, an ablation, is not drawn), at the same snapshot (streamer class), both tiers; the number under each group is its evaluation-set *median* κ (Tables 2 and 3), and the dashed lines mark the truth’s outward, inward and net levels. Learned codes reproduce the truth’s gross split with too little net; top- K and error-bounded codes cancel far more of their gross than the truth, by inflating the gross or collapsing the net or both; the supervised Fourier pins the net onto the dashed line without repairing its gross. The VAE group is one decode draw (see the draw band above).

Readout agreement. Measured on the deployed banks (the fitted per-snapshot atom sets): the per-mode potential agrees with the rendered pipeline at median reL1 $5.5\text{--}6.5 \times 10^{-6}$ (fp64 reference; the pipeline’s own fp32 floor is 7×10^{-6}), and the flux ratio is 1.0000 on every checked snapshot, worst $|\text{ratio} - 1|$ 2.8×10^{-5} over 108 snapshots. Table 2’s 10^{-4} floor sits just above this limit, which is why entries below it tie there; the polish itself takes the banded median to $\approx 2 \times 10^{-7}$, below it.

Query cost. Measured on an idle GH200 (medians of 30): a latent flux query costs 0.081 ms against 8.1 ms rendered ($100\times$), the one-off operator assembly costs 14.95 s for three k_y (the equivalent of about 1.9k rendered queries, fewer than the fit alone uses), and the flux gradient is one matvec ($244\times$ against autograd through a render proxy). Pre-assembly does not speed a full-field decode, which costs the same as rendering. The gains are for reduced observables queried repeatedly at frozen geometry, and above all for supervision, where the direct calculation and the constant Hessian matter more than speed. The full fit costs a median 270 s per snapshot at $N=9216$ on a GH200.

Sign flips across the evaluation set. For every scored reconstruction the evaluation records the net energy flux of the reconstruction and of the ground truth; comparing their signs per snapshot gives the wrong-sign counts of §3 (recomputed from the raw scoring rows). At $302\times$: db4 and bior4.4 wrong on 39/39 snapshots, Haar 35/39, db8 3/39; at $142\times$: db4 and bior4.4 38/39, Haar 17/39, db8 1/39; ZFP 5/39 and 1/39; SZ3 33/39 and 0/39; autoencoder, VAE, VQ-VAE, both neural fields and PCA, 0/39 at every measured budget: the learned and modal baselines err in magnitude, while the top- K wavelet codes err in sign. The Fourier pair and the objective-fitted Haar keep the banded net positive on every snapshot (39/39 at all four Fourier cells; 78/78 for Haar after the polish; banded-net instrument).

Wavelet variants. Table 2 keeps bior4.4, the best-field wavelet at $142\times$ and within 0.04 dB of db4 at $302\times$; the other three bases, scored identically on the same evaluation set:

Table 3: The remaining wavelet top- K variants (evaluation-set medians; wrong-sign counts as in the sign-flip paragraph above; κ as in Table 2, ground truth 1.89).

basis	CR	PSNR _f ↑	PSNR _φ ↑	flux relL1 ↓	κ	wrong sign
Haar	302.2×	36.20	−7.95	1.0150	9.22	35/39
db4	302.3×	36.46	−12.36	1.1497	20.2	39/39
db8	301.9×	34.09	−11.40	0.9020	23.0	3/39
Haar	142.2×	38.44	−7.12	0.9942	15.9	17/39
db4	142.2×	38.65	−9.21	1.1238	22.8	38/39
db8	142.2×	36.48	−8.43	0.8497	14.2	1/39

The field-objective ablation. Table 2’s GyroSplat (field objective only) rows run the pipeline’s own floor recipe (global normalization, random placement, field-only descent) with the carrier budget used instead for whole extra plain 16-real atoms (4,608 atoms = 73,728 reals at 302× and 9,792 = 156,672 at 142×, the noky accounting) at fp32 (78 fits, 0 gate failures). It reaches the comparison’s best field at both tiers while its potential sits ~19 dB below ours and its flux at 0.25/0.15. Its κ (1.80/1.79, IQR within [1.73, 1.88]; 0/39 sign flips) shows that localized atoms carry near-truth gross anatomy even observable-blind, that is, with no observable term in the fit. The anatomy comes with a dense fit (every densely fitted family, latents, PCA and both splat rows, prints κ 1.57–2.05, every top- K or error-bounded code 4.96–29.3), the net from the supervision. Table 5’s floor row (39.30 at the 17-real count) is its within-budget cousin.

The Fourier pair. Table 2’s Fourier top- K baseline keeps $K=78,336$ coefficients, the tier’s stored-value count under the same convention as the wavelet top- K rows, with its 82 KiB position-map floor disclosed above. Untouched by our pipeline, it reads 37.39/26.72/0.085 with κ 5.6. The supervised row keeps fewer coefficients, $K=68,799$, chosen so its fp32 values (537 KiB) *plus* position map (74 KiB) stay within the analytic row’s self-contained 612 KiB; positions are counted on both sides, as our atoms carry theirs inside the 17 N (39 snapshots, 0 gate failures both runs).

The supervised refit. The refit goes through the direct readouts. The loss is $\text{mse}_f/\text{mse}_f^0 + 0.5 \text{mse}_\phi/\text{mse}_\phi^0 + 0.5 \text{L1}_Q/\text{L1}_Q^0$ (superscript 0: each term’s value at init), Adam [19] 3×10^{-3} with cosine decay, 400 steps, then the pipeline’s own amplitude polish (the direct solve plus eight ridge-anchored Gauss–Newton steps on the flux).

What the refit shows. The flux collapses to $<10^{-4}$ at the median already under first-order descent (worst snapshot 0.0085) and on every snapshot after the polish, with field and potential each moving less than 0.01 dB on every snapshot. The refit leaves the field score where the unsupervised top- K put it: supervised through the direct readout, the flux column follows the target for any basis rich enough to span it, so it measures the supervision rather than the structure. The supervised row reads 37.16/27.11/ $<10^{-4}$ with κ 5.0 against our 38.25/28.71/ $<10^{-4}$ with κ 1.81: at equal total storage and equal supervision, the margins are 1.1 dB of field, 1.6 dB of potential, and a gross-to-net ratio at the truth’s 1.89 where Fourier still over-cancels 2.6×. The supervision constrains only the fifteen banded net values; the split into gross outward and inward weight is left free, and descent barely moves it (κ 5.8 at the supervised row’s own unsupervised start, 5.0 after; the plain baseline sits at 5.6). Fig. 3 shows the anatomy on one snapshot: the Fourier fit matches the net heat flux exactly, but its outward transport is twice and its inward transport nearly five times the true values. In contrast, GyroSplat reproduces both the outward and inward contributions more accurately.

The 302× pair. The 302× pair repeats the construction at the tighter budget ($K=36,864$ at the value count; $K=31,926$ with a 38.5 KiB map inside the 288 KiB): the baseline reads 35.90/22.60/0.19 (κ 7.0); supervised, the flux median is again $<10^{-4}$ (six snapshots sit above it, up to 0.0066) and the margins grow to 2.05 dB of field and 3.71 dB of potential (rows in Table 2).

The Haar boundary. The transfer has a measured boundary. The same chain on a *Haar* code at its own fair byte budget ($K=55,825/121,875$ kept values) leaves the flux untouched under first-order descent (1.00 before and after Adam, both budgets; the basis, not the optimizer) and the second-order polish pins all 15 banded values below 10^{-4} only by collapsing the field, 33.9 $\rightarrow$ 16.6 dB at the 302× budget and 35.5 $\rightarrow$ 26.4 at 142×, while κ *worsens* (10.6 $\rightarrow$ 14.5). Supervision through direct readouts is not sufficient; the retained support must be able to express the flux structure at

small field cost, which Fourier’s does, Haar’s does not, and the atoms do natively. The chain itself requires a code linear in its stored coefficients (§2); the neural field is not, so its physics stage supervises through its own render path instead.

Wrong-instance control and conditioning. The wrong-instance control (24-snapshot evaluation set, $N=9216$): co-fit targets drawn from a different snapshot of the same regime leave the flux at 0.62 against that control set’s 0.44 no-observable floor and drive the potential from the 21.8 dB floor to 7.3 dB, 14.5 dB below doing nothing, at or below floor on 24/24 snapshots, while field PSNR drops only ~ 4 dB. At a 512-atom stress budget, an order below deployed, 76% of the high- $|k_x|$ band is unreconstructed (relative ℓ_2 error 0.76 vs rank-4 SVD 0.264; radial correlation length ≈ 2 grid cells); separately, Φ is numerically full-rank on 5 tested snapshots ($\sigma_{\min}/\sigma_{\max} = 2.5 \times 10^{-7}$), so the potential map has no direct kernel, though its smallest singular directions sit at the fp32 floor. That conditioning is also the scale of the field-to-potential collapse in Table 2: a linear solve with $\sigma_{\max}/\sigma_{\min} \approx 4 \times 10^6$ can amplify field error by up to ~ 130 dB into ϕ , which is how 36–39 dB fields print -8 to -15 dB potentials.

B The readouts, step by step

This section walks through §2’s identities, with explicit shapes at every step. Two notes on the atom itself: the lower-triangular, positive-diagonal shape factors make every real parameter vector a valid atom, and the per-atom velocity-space factorization is not restrictive, since a sum of separable atoms can still represent coupled structure. Flattened, one snapshot is $\text{vec}(\mathbf{x}) \in \mathbb{C}^M$ with $M = 32 \cdot 8 \cdot 16 \cdot 85 \cdot 32 = 11,141,120$, and at frozen geometry the decode is the linear map $\hat{\mathbf{x}} = \Psi(\theta)a$ with $\Psi \in \mathbb{C}^{M \times N}$, whose column i is atom i rendered at unit amplitude.

The observables as maps of any field. The two observables are properties of the field itself, before any representation enters. For an arbitrary field $\mathbf{x}$, quasineutrality is a linear map $\mathcal{Q} : \mathbb{C}^M \rightarrow \mathbb{C}^P$ to the potential, with $P = 16 \cdot 85 \cdot 32 = 43,520$ (the spatial grid; velocity is integrated out), and the flux is a sesquilinear pairing $Q(k_y) = \mathcal{B}_{k_y}(\mathbf{x}, \phi)$ of a velocity moment of $\mathbf{x}$ with the $E \times B$ drift from ϕ (per mode $\propto i k_y \phi$). Since $\phi = \mathcal{Q}(\mathbf{x})$ is itself linear in $\mathbf{x}$, the flux is a quadratic functional of the field. Both statements hold for the ground truth as much as for any reconstruction.

Substituting the decode. Putting $\hat{\mathbf{x}} = \Psi a$ into the two maps and using linearity,

$$\begin{aligned}\hat{\phi} &= \mathcal{Q}(\Psi a) = \sum_i a_i \mathcal{Q}(\psi_i) = \Phi a, \\ \hat{Q}(k_y) &= \mathcal{B}_{k_y} \left(\sum_i a_i \psi_i, \sum_j a_j \mathcal{Q}(\psi_j) \right) = \sum_{i,j} \bar{a}_i a_j [A_{k_y}]_{ij} = a^H A_{k_y} a,\end{aligned}$$

with $\Phi = [\mathcal{Q}(\psi_1) \cdots \mathcal{Q}(\psi_N)] \in \mathbb{C}^{P \times N}$ and $[A_{k_y}]_{ij} = \mathcal{B}_{k_y}(\psi_i, \mathcal{Q}(\psi_j))$, Hermitized, which keeps $\hat{Q}$ real. Column i of Φ is the potential field of atom i at unit amplitude, and entry (i, j) of A_{k_y} is the flux contribution of the atom pair: atom i ’s distribution side paired with atom j ’s potential side. There is one $N \times N$ Hermitian matrix per scored flux mode ($k_y = 1, \dots, 15$; the zonal mode carries no flux). A single atom alone carries no net flux (the representation’s flux diagonal is zero, measured), so the whole flux lives in the off-diagonal interference, which is precisely what the matrices store.

Per-atom assembly. Once per frozen geometry, each atom is rendered alone (a full 5D tensor at unit amplitude) and passed through the reference integral chain a single time. Its potential response $\mathcal{Q}(\psi_i) \in \mathbb{C}^{43,520}$ becomes a column of Φ , and its per-mode flux-side responses (each living on the (s, x) slice of $16 \cdot 85 = 1,360$ points) are stored. The entries of A_{k_y} are inner products of these stored responses, so no pair is ever rendered, and $\text{rank } A_{k_y} \leq 2 \cdot 1,360 = 2,720$, which is why A is used through its thin factors rather than as a dense block. Rendering therefore happens once per atom at assembly time instead of once per optimizer step.

Table 4: Shape summary of the readout construction.

object	shape	count	meaning
snapshot $\mathbf{x}$	$(32, 8, 16, 85, 32), \mathbb{C}$	$M = 11,141,120$	distribution on the 5D grid
atom render ψ_i	same as $\mathbf{x}$	one per atom	unit-amplitude atom field
$\Psi(\theta)$	$M \times N$	–	decode matrix, $\hat{\mathbf{x}} = \Psi \mathbf{a}$
$\mathbf{a}$	$N, \mathbb{C}$	$N = 4,337 / 9,216$	stored amplitudes
potential ϕ	$(16, 85, 32), \mathbb{C}$	$P = 43,520$	spatial grid only
Φ	$P \times N$	–	column i = potential of atom i
A_{k_y}	$N \times N$, Hermitian	15 (modes 1..15)	$[A]_{ij}$ = pair (i, j) flux
atom parameters	17 reals	per atom	§2

Input: snapshot $\mathbf{x}$; atom budget N ; carrier budget fraction $f=0.25$.

- 1 **normalize:** split the zonal ($k_y=0$) channel from the fluctuating rest, scale each separately (both inverted at decode).
why: under one global scale the optimizer spends its budget on the zonal channel, which carries most of the power and none of the flux.
- 2 **place:** sample atom centers $\propto |\mathbf{x}|$; **solve** amplitudes by the direct ridge system (the field term is linear least squares in $\mathbf{a}$).
why: atoms are local, so placement bounds what the fit can represent; the ridge solve starts each placement at its best amplitudes.
- 3 **warm up geometry:** AdamW on centers and Cholesky factors, robust field loss only, cosine schedule.
- 4 **physics fine-tune:** 40 geometry epochs through the reference integral operators, field + potential terms on, *flux term deliberately off*.
why: anisotropy aligns to shapes the ϕ -operator rewards; withholding the flux keeps every stage to here transport-blind, so later flux gains are attributable.
- 5 **add carriers:** place carrier atoms beside the base atoms, taking a fraction f of the atom budget; solve their amplitudes on the residual.
- 6 **masked co-fit:** per epoch, base rows step on $\nabla(\text{field} + \phi)$ and carrier rows on $\nabla(\text{banded } Q)$; core is the ϕ -aware amplitude solve.
why: each row receives gradient from one term only, so the terms cannot trade against each other; the plain field solve is the measured trap of Table 5 (field_solve).
- 7 **flux polish:** geometry frozen, 8 Gauss–Newton steps of the amplitudes on $Q(k_y)$, ridge-anchored to the initial solve. *why: Q is quadratic in $\mathbf{a}$, so the Jacobian is available in closed-form and the Hessian constant; the ridge keeps the field/ ϕ fit the earlier stages paid for.*
- 8 **write out:** store the reals at fp32; the stored count is the printed budget.

Figure 5: The fitting pipeline as pseudocode, one deployed stage per numbered step (the cumulative stages of Table 5, floor through the flux polish; floor is step 3 alone at global normalization and random placement).

C The fitting pipeline

The fit is a fixed cumulative pipeline (Fig. 5). Each stage adds one lever, so each is ablatable in isolation, and Table 5 scores every stage and every one-lever ablation on the full evaluation set.

Stage notes. Details the pseudocode does not carry: the warmup optimizer is AdamW [25] on a Huber field loss. The Huber choice is measured, not assumed: refitting all 39 snapshots with a plain MSE field term (fp16-packed stage) reads 38.15/26.75/0.0027 at $302\times$ and 38.76/28.26/0.0044 at $142\times$ against the packed Huber rows’ 37.70/27.15/0.0018 and 38.24/28.28/0.0030, so no column moves by more than 0.6 dB and the loss choice is not what separates the analytic rows from the L_2 -trained baselines. The deployed carriers sit at binormal modes 7–13; the written file stores 73,728/156,669 reals at fp32 (within four reals of $17N$), with the fp16 cast kept as the `s7_pack` ablation. Stages up to the co-fit differentiate through the reference render chain; the polish alone uses the assembled quadratic form.

Reading the stages. The pipeline is not monotone. The observable-blind stages *worsen* the flux and move the potential up and down, and each observable improves only once its own term enters the

Table 5: The pipeline at 302 $\times$: cumulative stages (top) and one-lever ablations (below), medians over the 39 evaluation snapshots, canonical data root. The deployed row reproduces Table 2’s GyroSplat row; the paragraphs below discuss each stage and ablation.

stage / ablation	PSNR _f $\uparrow$	PSNR _{ϕ} $\uparrow$	flux relL1 $\downarrow$
floor (global norm, random placement)	39.30	7.31	0.2519
+ zonal-split normalization	38.35	12.29	0.3328
+ importance placement, exact amplitude init	39.04	10.29	0.3815
+ geometry warmup (field only)	37.99	8.21	0.3327
+ physics fine-tune (ϕ on, flux off)	38.13	25.25	0.2819
+ carriers	37.85	22.09	0.2519
+ ϕ -aware observable co-fit	37.71	27.30	0.0001
+ flux Gauss–Newton polish (deployed)	37.70	27.30	$< 10^{-4}$
co-fit $\rightarrow$ field-optimal solve (field_solve)	37.89	8.75	0.2665
polish without co-fit (gn_nocofit)	37.83	22.35	$< 10^{-4}$
no carrier DOF, equal bytes, to fine-tune (noky)	38.17	26.06	0.2863
random placement, to init stage (random_init)	39.13	16.12	0.2425
random placement, end-to-end (random_full)	36.63	25.37	0.0028
no exact amplitude init, to init stage (no_warmstart)	35.93	−2.65	0.6536
flux term on at the fine-tune (flux_on)	37.54	20.84	0.4157
every atom a carrier, to carrier stage (all_gabor)	38.84	25.74	0.2214
every atom a carrier, end-to-end (all_gabor_full)	37.68	−12.39	0.0061
carrier fraction 0.40, to carrier stage (frac40)	37.60	20.02	0.2673
carrier fraction 0.60, to carrier stage (frac60)	37.06	16.91	0.2557
quantize to fp16 (0.5 $\times$ bytes) (s7_pack)	37.70	27.15	0.0018
fp32 at the fp16 byte cost ($N=2168$) (fp32match)	36.86	24.60	$< 10^{-4}$
no carriers, un-masked flux chain (noky_full)	36.35	−17.01	0.0226
deployed atoms, un-masked flux chain (base_gnall)	35.72	−15.33	0.0203

objective. The last transport-blind stage (no flux term yet) sits at flux relL1 0.25 at 302 $\times$ and 0.18 at 142 $\times$ (the ϕ fine-tune above it: 0.28, IQR 0.23–0.34), and the collapse to 10^{-4} happens at the co-fit, where the direct readout enters the objective. Against the observable-blind floor the deployed row gives up 1.6 dB of field PSNR, 0.3 of it to the physics columns (warmup vs deployed) and the rest to the floor’s normalization and placement freedom.

Reading the ablations. Each ablation removes one lever. The field-optimal amplitude solve (field_solve) improves the field while collapsing the potential by 18.6 dB and leaving the flux at its observable-blind value. The polish without the co-fit (gn_nocofit) still collapses the flux but loses 5.0 dB of potential. Skipping the exact amplitude initialization costs 3.1 dB field and 13 dB potential at the init stage. Random placement scores *above* importance placement at the init stage yet is worse end-to-end on every column (−1.1 dB field, −1.9 dB potential, flux 0.0028 against the deployed $< 10^{-4}$), so the placement only shows its value through the later stages. Switching the flux term on early (flux_on) is worse on every metric, including the flux itself (0.42 vs 0.28).

Carriers and masking. The atom set has two groups. Base atoms carry $k_{y,i} = 0$ and are fitted first; carrier atoms carry $k_{y,i} > 0$, are added afterwards, and take a fraction f of the atom budget ($f=0.25$ deployed). The mask decides which loss reaches which group during the co-fit. The field and potential gradients update only the base atoms and the flux gradient updates only the carriers, so improving the flux cannot degrade the group that holds the field and the potential. The ablations measure both choices. Carrier fractions of 0.40 and 0.60 underperform the deployed 0.25 at the carrier stage, and the all-carrier bank, although better at that stage, collapses end-to-end (ϕ at −12.4 dB, flux 0.0061) because no protected base group remains for the masked co-fit. The two un-masked chains at the bottom of the table show the same failure. With and without the carrier parameter they are nearly identical, collapse the flux only to 0.02, and destroy the potential (−15 to −17 dB). The carrier’s value is therefore in the *masking*. Masked supervision reaches 10^{-4} flux with the potential intact at 27.3 dB, while the un-masked chain stops at 200 $\times$ more flux error and loses the potential entirely.

Bytes against precision. Quantizing the deployed row to fp16 (s7_pack, half the bytes) costs 0.15 dB potential and leaves 0.0018 flux, and the error is entirely the cast. At that same byte cost,

fp32 at half the atom count (fp32match) keeps the flux below 10^{-4} but cedes 0.8 dB field and 2.6 dB potential, so at fixed bytes more atoms beat more precision.

Why Gabor, and why this carrier layout. On sibling codecs of the same atom family the carrier necessity is measured directly: with the objective and budget fixed and only the atom type varied, plain Gaussians (broad or narrow) reach potential PSNR $-3.6/0.1$ dB where Gabor atoms reach 15.6/17.3 dB, and matching the Gabor tail-flux with plain Gaussians costs +300% bytes and still falls short. The carrier degree of freedom is also not discoverable by descent: with a free per-atom k_y under an energy objective, no atom ever develops a carrier (max k_y reached 0.16 against a target band at $k_y \approx 19\text{--}94$, even at $10\times$ learning rate), because the energy at high k_y is too small to produce gradient off $k_y=0$; carriers must be seeded. The layout sits on a plateau. A 576-configuration factorial over multi-group, harmonic-comb, co-located and structure-atom variants is within ± 0.4 dB of the deployed contiguous tail carriers; a harmonic comb is falsified on the data alone (the k_y spectrum is a broadband power law, so a comb covers 0.51 of the tail against 0.85 for free carriers); and co-locating carriers on one envelope is catastrophic per- k_y ($\phi -7.4$ dB) while nearly invisible in MSE, the same field/observable split the paper measures at family level. The deployed pipeline shows the complementary fact. Removing the carrier parameter entirely (16-real atoms, equal bytes) is *free* while the objective is transport-blind (noky in Table 5 matches the deployed fine-tune stage), because the carrier’s value is realized only through the flux-aware stages, whose gradient is masked to carriers by construction. (Scope: the modulation-necessity and layout controls above ran on sibling codecs of the same atom family at $N=1000\text{--}4302$.)

D Dispersion and per-class medians

Table 6: Dispersion behind Table 2: median [interquartile range] over the same 39 snapshots, re-computed from the raw per-snapshot scores. All medians reproduce the printed table. The flux IQRs of the analytic rows are disjoint from every observable-blind baseline’s, with a gap of nearly three orders of magnitude between the closest quartile edges (both objective-fitted rows share the 10^{-4} floor); the potential margin over the physics-informed neural field is *inside* quartile overlap and is claimed only as a median. Paired per-snapshot comparisons say the same: on flux the analytic row wins all 39 of 39 against every observable-blind row at both tiers (worst blind per-snapshot value 0.033 against our $< 10^{-4}$; sign test $p = 2^{-39}$ per pairing, $p \leq 2^{-27}$ clustered by the 27 source trajectories). The sign-flips column counts the snapshots, out of 39, on which the reconstruction’s net energy flux has the opposite sign to the ground truth’s; the banded-net check gives identical counts on every row it covers.

family	CR	PSNR _f ↑	PSNR _φ ↑	flux relL ↓	sign flips ↓
<i>CR ≈ 302×</i>					
GyroSplat (ours)	302.2×	37.70 [37.12, 38.39]	27.30 [26.14, 29.33]	$< 10^{-4}$ (all quartiles)	0
GyroSplat (field obj. only)	302.2×	39.38 [38.68, 40.18]	8.43 [4.28, 11.81]	0.2537 [0.1884, 0.3000]	0
Neural field (field obj.)	301.9×	37.39 [36.77, 37.89]	14.73 [11.78, 17.93]	0.1508 [0.1078, 0.1781]	0
Neural field (+physics)	301.9×	37.33 [36.71, 37.84]	26.50 [25.15, 29.46]	0.1249 [0.1000, 0.1445]	0
Autoencoder	302.2×	35.49 [34.86, 36.26]	15.42 [11.77, 18.39]	0.1048 [0.0784, 0.1515]	0
VAE	302.2×	34.33 [33.52, 35.35]	9.26 [6.04, 13.16]	0.1783 [0.1345, 0.2470]	0
VQ-VAE	302.2×	34.16 [33.42, 34.82]	12.91 [9.50, 14.52]	0.1938 [0.1644, 0.2264]	0
Wavelet (bior4.4)	302.7×	36.42 [35.80, 36.97]	−12.35 [−19.56, −3.86]	1.2317 [1.1597, 1.2661]	39
Wavelet (Haar)	302.2×	36.20 [35.53, 36.55]	−7.95 [−15.21, 0.85]	1.0150 [1.0101, 1.0168]	35
Wavelet (db4)	302.3×	36.46 [35.82, 36.97]	−12.36 [−19.51, −3.84]	1.1497 [1.1109, 1.1814]	39
Wavelet (db8)	301.9×	34.09 [33.72, 34.81]	−11.40 [−18.65, −2.12]	0.9020 [0.8873, 0.9156]	3
PCA	302.7×	35.08 [34.50, 36.06]	−2.54 [−8.68, 4.97]	0.4348 [0.3354, 0.6353]	0
ZFP	304.8×	32.72 [32.16, 33.13]	−5.61 [−12.59, 3.08]	0.9781 [0.9281, 0.9897]	5
SZ3	301.5×	33.42 [32.45, 34.27]	−15.34 [−24.20, −9.19]	1.2874 [1.0530, 1.4235]	33
Fourier top- <i>K</i>	302.2×	35.90 [35.34, 36.57]	22.60 [21.00, 26.57]	0.1899 [0.1550, 0.2278]	0
Fourier (+ our objective)	302.3×	35.65 [35.10, 36.36]	23.59 [22.09, 26.86]	$< 10^{-4}$ (all quartiles)	0
Haar (+ our objective)	302.2×	16.60 [12.62, 18.69]	−8.50 [−14.48, −2.24]	$< 10^{-4}$ (all quartiles)	0
<i>CR ≈ 142×</i>					
GyroSplat (ours)	142.2×	38.25 [37.64, 38.89]	28.71 [27.79, 30.85]	$< 10^{-4}$ (all quartiles)	0
GyroSplat (field obj. only)	142.2×	41.52 [40.64, 42.17]	9.92 [2.74, 14.23]	0.1469 [0.1232, 0.1707]	0
Neural field (field obj.)	142.0×	38.17 [37.65, 38.82]	14.66 [12.07, 20.01]	0.1199 [0.0912, 0.1530]	0
Neural field (+physics)	142.0×	38.12 [37.42, 38.49]	26.99 [26.06, 28.94]	0.0890 [0.0735, 0.1095]	0
Wavelet (bior4.4)	142.2×	38.94 [38.34, 39.57]	−9.86 [−16.55, −1.54]	1.1441 [1.1021, 1.2092]	38
Wavelet (Haar)	142.2×	38.44 [37.92, 38.96]	−7.12 [−13.29, 1.27]	0.9942 [0.9675, 1.0058]	17
Wavelet (db4)	142.2×	38.65 [38.24, 39.25]	−9.21 [−16.39, −0.59]	1.1238 [1.0652, 1.1506]	38
Wavelet (db8)	142.2×	36.48 [35.90, 37.04]	−8.43 [−14.14, 1.71]	0.8497 [0.8208, 0.8629]	1
PCA	143.4×	37.88 [37.33, 38.75]	1.69 [−1.05, 7.87]	0.3371 [0.2699, 0.4093]	0
ZFP	144.9×	35.29 [34.80, 36.17]	−3.39 [−6.40, 4.69]	0.7941 [0.7466, 0.8905]	1
SZ3	142.2×	38.36 [37.49, 39.28]	−11.93 [−21.54, −6.64]	0.5311 [0.4241, 0.5851]	0
Fourier top- <i>K</i>	142.2×	37.39 [36.60, 37.78]	26.72 [23.94, 29.79]	0.0850 [0.0741, 0.1037]	0
Fourier (+ our objective)	142.4×	37.16 [36.38, 37.53]	27.11 [25.05, 30.15]	$< 10^{-4}$ (all quartiles)	0
Haar (+ our objective)	142.2×	26.40 [22.40, 28.40]	−6.94 [−12.66, 2.66]	$< 10^{-4}$ (all quartiles)	0

Table 7: Per-class medians at $CR \approx 302 \times$ (12 streamer / 12 mixed / 15 strong-zonal), same snapshots and scorer. Streamer snapshots are the hardest class for our fit on field and potential (our flux column stays below 10^{-4} in every class) and for four of the five learned rows on the flux, while the saturated wavelet rows, PCA and ZFP scramble that ordering. Rows follow Table 2’s order and cover every Table 1–2 method at $302 \times$.

family	streamer (12)			mixed (12)			strong zonal (15)		
	PSNR _f	PSNR _φ	relL1	PSNR _f	PSNR _φ	relL1	PSNR _f	PSNR _φ	relL1
GyroSplat (ours)	37.24	26.44	$< 10^{-4}$	38.10	26.51	$< 10^{-4}$	37.70	29.52	$< 10^{-4}$
GyroSplat (field obj. only)	39.64	5.12	0.3217	39.47	9.61	0.2617	39.26	8.57	0.1969
Neural field (field obj.)	37.42	16.91	0.1740	37.34	12.84	0.1569	37.45	14.73	0.1086
Neural field (+physics)	37.19	24.87	0.1044	37.26	26.66	0.1352	37.33	29.81	0.1252
Autoencoder	34.15	15.86	0.1515	35.77	13.71	0.1111	36.23	16.01	0.0778
VAE	32.39	10.67	0.2655	34.64	10.31	0.1834	35.35	9.13	0.1336
VQ-VAE	33.30	11.55	0.2283	34.05	13.34	0.2173	34.80	12.69	0.1614
Wavelet (bior4.4)	36.88	−3.89	1.2539	36.25	−11.26	1.1874	36.25	−16.21	1.2346
Wavelet (Haar)	36.39	0.85	1.0145	35.86	−6.93	1.0163	36.00	−12.20	1.0125
Wavelet (db4)	36.77	−3.84	1.2016	36.26	−11.52	1.1436	36.37	−15.80	1.1453
Wavelet (db8)	34.62	−2.12	0.9009	33.95	−10.12	0.8925	34.08	−14.59	0.9044
PCA	34.71	4.21	0.4771	34.77	−4.27	0.5751	35.40	−4.39	0.3471
ZFP	32.40	2.99	0.9490	32.58	−4.54	0.9898	33.08	−8.40	0.9703
SZ3	34.27	−13.26	1.0571	33.67	−15.80	1.3191	32.81	−16.32	1.3143
Fourier top- K	35.58	20.17	0.2212	36.04	22.97	0.2107	36.42	26.69	0.1524
Fourier (+ our objective)	35.32	22.03	$< 10^{-4}$	35.77	24.13	$< 10^{-4}$	36.20	27.29	$< 10^{-4}$
Haar (+ our objective)	17.22	−2.65	$< 10^{-4}$	15.37	−7.16	$< 10^{-4}$	17.24	−9.55	$< 10^{-4}$